Consumer sentiment and business cycles: A Neimark-Sacker bifurcation scenario

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Abstract

We seek to demonstrate that consumer sentiment may create fluctuations in economic activity. Our nonlinear discrete-time model possesses, for instance, a Neimark-Sacker bifurcation, after which a stable steady state is replaced by (quasi-)periodic motion. Countercyclical interventions to stabilize the economy may even produce complex (chaotic) business cycles.

Keywords

Business cycles; Consumer sentiment; Fiscal policy; Neimark-Sacker bifurcation; Chaos

JEL classification

E12, E32, E62

1 Introduction

When it comes to explaining the causes of business cycles, consumer sentiment has received only scant attention until now. Although consumption expenditures are less variable than investment expenditures, they account for a much larger part of national income. Empirical evidence (Carroll et al. 1994, Souleles 2004, Doms and Morin 2004) suggests that consumer sentiment is a biased mirror of economic reality and determines household consumption behavior, thus confirming Keynes' suspicion that consumer "attitudes" and "animal spirits" may cause fluctuations in economic activity.

Inspired by these observations, we develop a novel business cycle model in which the agents' consumption expenditures depend on their sentiment. In particular, the agents optimistically (pessimistically) consume a higher (lower) fraction of their income when national income increases (decreases). Our two-dimensional nonlinear model has a unique steady state which may, for instance, become unstable if autonomous expenditures exceed a critical threshold. Due to a supercritical Neimark-Sacker bifurcation, however, global stability may continue in the form of an attractor which is a sequence of points lying on a closed curve. Countercyclical interventions of policy makers may turn the (quasi-) periodic behavior of national income into complex (chaotic) business cycles.

Nonlinear business cycle models are discussed in depth by Puu (1989), Hommes (1991), Medio (1992), Day (1999), Rosser (2000) and Puu and Sushko (2006). These models are mainly concerned with nonlinear investment functions in the tradition of Kaldor, Hicks or Goodwin and may generate interesting endogenous dynamics. We continue as follows. In section 2, we develop our model. In section 3, we present our analytical results and in section 4, we numerically illustrate them. Section 5 concludes.

2 The model

To make matters as simple as possible, the underlying framework of the present paper is the well-known Keynesian multiplier model of the real sector. National income Y at time step t is written as

$$Y_t = C_t + G_t . \tag{1}$$

The consumption expenditures are partially autonomous and partially dependent on the last period's income. Instead of assuming that agents consume a constant fraction of their income, we argue that their consumption behavior is influenced by their sentiment. Consumption expenditures are thus formalized as

$$C_t = a + S_{t-1} Y_{t-1} , \tag{2}$$

where $a > 0$ stands for the autonomous expenditures. The fraction of income which is consumed by the agents is given by the S-shaped function

$$S_{t-1} = b + \frac{c}{1 + Exp[-(Y_{t-1} - Y_{t-2})]} . \tag{3}$$

In a steady state, the agents consume $0 < b + 0.5c < 1$ of their income. When income (strongly) decreases, the agents become pessimistic and consume $0 < b < 1$ of their income. When income (strongly) increases, they become optimistic and consume $b < b + c < 1$ of their income. Note that Souleles (2004) finds, in fact, that higher consumer confidence is correlated with less saving, consistent with precautionary motives and increases in expected future resources.

Policy makers often seek to stabilize the economy. Following Baumol (1961), we consider the case in which the government determines to offset income trends by deficit spending when income has just been falling and by collecting a budget surplus when income has been rising. Such a trend-offsetting rule may be formalized as

$$G_t = d(Y_{t-2} - Y_{t-1}), \tag{4}$$

where $0 \le d \le 1$ is the policy maker's control parameter.

Combining (1) to (4) reveals that

$$Y_t = a + (b-d)Y_{t-1} + dY_{t-2} + \frac{cY_{t-1}}{1 + Exp[-(Y_{t-1} - Y_{t-2})]}, \tag{5}$$

i.e. the recurrence relation which determines national income is a two-dimensional nonlinear difference equation.

3 Analytical results

In the appendix, we prove the following results:

(R1) Our model has a unique fixed point $\overline{Y} = a/(1-(b+0.5c))$, which is equal to the well-known Keynesian multiplier solution.

(R2) The fixed point is locally asymptotically stable if

$$\frac{2((b+0.5c)^2 - 1) + 4d(1-b-0.5c)}{c} < a < \frac{4(1+d)(1-b-0.5c)}{c}.$$

(R3) At the lower stability frontier, a flip bifurcation may occur, i.e. the steady state becomes unstable and a stable period-two cycle emerges.

(R4) At the upper stability frontier, we observe a supercritical Neimark-Sacker bifurcation, i.e. the steady state becomes unstable and a closed curve emerges on which both quasiperiodic and periodic cycles may lie.

Let us briefly discuss the last results which we find particularly interesting. A Neimark-Sacker bifurcation implies the onset of (quasi-)periodic motion. Business cycles may thus at least partially be driven by consumer sentiment. In our case, a high marginal propensity to consume may destabilize the system. The critical value for a

which ensures local asymptotic stability of the steady state decreases with b and c. By contrast, an increase in d shifts the stability boundary upwards. At least at first sight, this may be regarded as good news for policy makers.

4 Numerical illustration

Figure 1 shows a bifurcation diagram in which autonomous expenditures are increased from 19 to 21 in 200 discrete steps. For each value of a, we plot 100 observations (after omitting a longer transient period). Bifurcation diagrams are a powerful tool to illustrate the dynamics of nonlinear models. The other parameters are given as $b = 0.45$, $c = 0.1$, and $d = 0$. As one would expect, an increase in autonomous expenditures drives the steady-state level of national income upwards. At $a = 20$, however, a Neimark-Sacker bifurcation becomes visible and (quasi-)periodic behavior sets in.

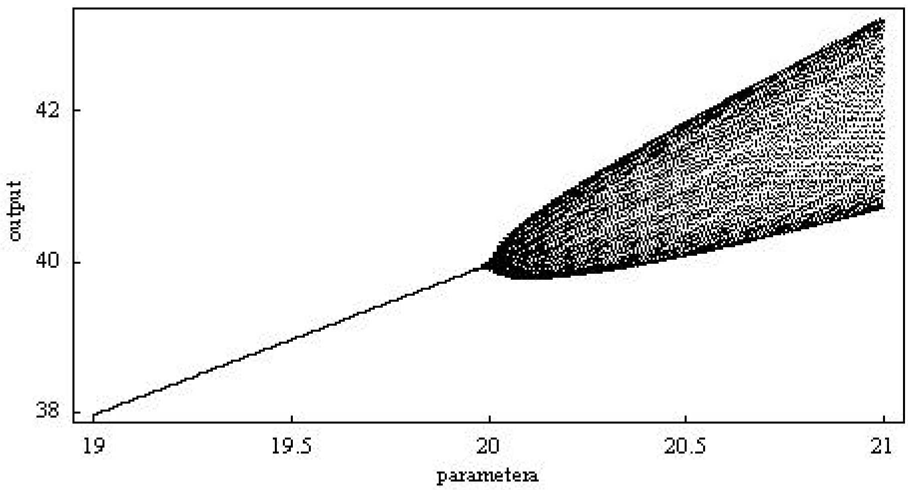

Figure 1: Bifurcation diagram for autonomous expenditures. Parameter a is increased from 19 to 21 in 200 discrete steps. For each value of a, 100 observations are plotted (after erasing a longer transient). The remaining parameters are $b = 0.45$, $c = 0.1$, and $d = 0$.

This is further illustrated in figure 2, which displays the dynamics of the model in both time domain (top) and phase space (bottom). Now we set $a = 250$, $b = 0.45$ and $c = 0.1$ and vary the policy maker's control parameter between $d = 0$ (left), $d = 0.6$ (central) and $d = 0.8$ (right), respectively. Without government interventions, we observe quite regular business cycles. The model works roughly as follows: Suppose that national income increases. As a result, the agents are optimistic and consume a larger fraction of their income. This obviously creates an upswing. But the expansion automatically loses its momentum when the marginal propensity to consume reaches its maximum value. Optimism then vanishes and a recession sets in.

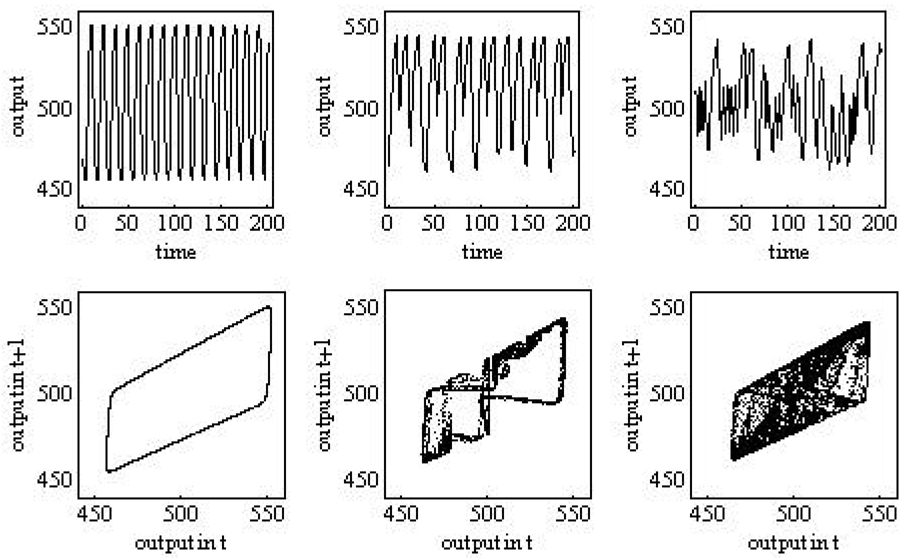

Figure 2: The evolution of national income in the time domain and in phase space for different values of the policy maker's control parameter. The parameters are $a = 250$, $b = 0.45$, $c = 0.1$, $d = 0$ (left), $d = 0.6$ (central) and $d = 0.8$ (right).

Countercyclical policies may change the dynamics of the model in a nontrivial way. For $d = 0.8$, for instance, the evolution of national income is quite complex (chaotic) and in phase space, a strange attractor appears. On the other hand, one should recall that when the system is not too far away from the Neimark-Sacker bifurcation, an increase in d may ensure that the fixed point is at least locally asymptotic stable.

5 Conclusions

Empirical evidence suggests that consumer sentiment may play a crucial role in the understanding of business cycles. Our goal is to pick up on the empirical literature and to develop a business cycle model which explicitly takes into account that the agent's consumption expenditures may depend on economic circumstances. We find that if consumers condition their consumption expenditures on the change of national income, endogenous fluctuations of economic activity may set in. In particular, there exists a critical level for autonomous expenditures after which the model's steady-state solution loses stability and the evolution of national income becomes (quasi-)periodic. Countercyclical policy interventions may transform regular motion into chaotic motion.

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Appendix

By introducing an auxiliary variable $Z_t = Y_{t-1}$, we may rewrite (5) as a first-order system in (Y_t, Z_t), i.e.

$$Y_t = a + (b-d)Y_{t-1} + dZ_{t-1} + \frac{cY_{t-1}}{1 + Exp[-(Y_{t-1} - Z_{t-1})]} \tag{6}$$

and

$$Z_t = Y_{t-1}. \tag{7}$$

Recall that $a > 0$, $0 < b, c < 1$, $b + c < 1$ and $0 \le d \le 1$. Inserting $\overline{Y} = \overline{Z} = Y_{t-1} = Z_{t-1}$ in (6) and (7) reveals that the unique steady state of our model is determined by

$$\overline{Y} = \overline{Z} = \frac{a}{1 - b - 0.5c}. \tag{8}$$

The Jacobian matrix of our model may be expressed as

$$J(Y,Z) = \begin{pmatrix} b - d + \dfrac{c\,(1 + Exp[-Y+Z]) + c\,Y\,(Exp[-Y+Z]}{(1 + Exp[-Y+Z])^2} & d - \dfrac{c\,Y\,Exp[-Y+Z]}{(1 + Exp[-Y+Z])^2} \\ 1 & 0 \end{pmatrix}. \tag{9}$$

Calculated at the steady state, the Jacobian matrix simplifies to

$$J(\overline{Y}) = \begin{pmatrix} b - d + \dfrac{2c(b + 0.5c - 0.5a - 1)}{4b + 2c - 4} & d + \dfrac{ac}{4b + 2c - 4} \\ 1 & 0 \end{pmatrix}, \tag{10}$$

with trace

$$tr\ J(\overline{Y}) = b - d + \frac{2c(b + 0.5c - 0.5a - 1)}{4b + 2c - 4} \tag{11}$$

and determinant

$$det\ J(\overline{Y}) = -d - \frac{ac}{4b + 2c - 4}. \tag{12}$$

Necessary and sufficient conditions guaranteeing that a fixed point of a two-dimensional map is locally asymptotically stable are (i) $1 + tr\ J(\overline{Y}) + det\ J(\overline{Y}) > 0$, (ii) $1 - tr\ J(\overline{Y}) + det\ J(\overline{Y}) > 0$, and (iii) $1 - det\ J(\overline{Y}) > 0$, respectively. After some rearrangements, we obtain

$$a > \frac{2((b+0.5c)^2-1)}{c} + \frac{4d(1-b-0.5c)}{c}, \tag{13}$$

$$(b+0.5c-1)^2 > 0, \tag{14}$$

and

$$a < \frac{4(1+d)(1-b-0.5c)}{c}. \tag{15}$$

A flip bifurcation may occur when the first inequality is violated (Medio and Lines 2001). The first term on the right hand side of (13) is always negative while the second term is always positive. Note that there are parameter combinations for which the critical value of *a* is indeed positive.[1]

Stability will also be lost at the value for which (15) becomes an equality, the critical value for the Neimark-Sacker bifurcation (Medio and Lines 2001). The numerical evidence presented in section 4 indicates that the Neimark-Sacker bifurcation is of the supercritical type, i.e. crossing the bifurcation boundary is followed by the appearance of an invariant circle.

[1] For instance, if a=0.1, b=0.5, c=0.49 and d=0.95, we observe a stable period-two cycle. In this case, the critical value for a is a=0.1613.