

Speculative dynamics, feedback traders and transaction taxes: A note

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1. Introduction

Financial markets are extremely volatile and frequently display severe bubbles and crashes. Such price dynamics is obviously caused by the behavior of the market participants. Keynes (1936) already observed that many traders are subject to waves of optimistic and pessimistic sentiment. Prices may therefore change violently as a result of a sudden shift of market opinion. Moreover, many investors are concerned with outwitting the crowd. What matters is not what an investment is worth but what the crowd thinks how the crowd will evaluate it. Keynes was convinced that pure *laissez-faire* capitalism does not fulfill its social purpose. To shift the predominance of destabilizing short-term speculation over stabilizing long-term investment, regulators of financial markets should thus impose transaction taxes. Keynes' proposal received renewed attention when Tobin (1978) suggested the introduction of a uniform tax of around 1 percent on all currency transactions in order to placate foreign exchange dynamics. Note that even small tax rates may have a strong impact on high-frequency trading. A 0.5 percent transaction tax translates into a 12 percent annual rate on a month trip or a 365 percent tax on a one day round trip. Clearly, the longer the horizon of the investor, the lower the tax burden.

Although transaction taxes are vividly debated in the popular media, (mathematical) academic scrutiny has remained scant (for general surveys see, e.g. Eichengreen et al. 1995, Haq et al. 1996). Inspired by the chartist-fundamentalist approach (Day and Huang 1990, Brock and Hommes 1998, Lux and Marchesi 2000, Rosser et al. 2003) we develop a stylized asset pricing model with heterogeneous boundedly rational speculators to explore the consequences of transaction taxes. Within our model, prices adjust due to the orders of the market players. While chartists extrapolate past price changes into the future, fundamentalists believe that prices converge towards the asset's intrinsic value. The law of motion of the asset price turns out to be a second-order linear difference equation. As is well known, such a system is capable of generating cyclical motion. Concerning transaction taxes, we analytically show the following results. First, the long-run equilibrium price, i. e. the intrinsic value, is independent of transaction taxes. Second, transaction taxes enlarge the area in which price fluctuations are stable. Third, transaction taxes diminish the amplitude of price oscillations.

We continue as follows. Section 2 presents a simple linear asset pricing model with chartists and fundamentalists. In section 3, we derive our analytical results and in section 4, we numerically illustrate our findings. The last section concludes the note

2. A stylized asset pricing model

Let us briefly motivate the behavioral background of our model. Agents are regarded as boundedly rational in the sense of Simon (1955): Information is in general incomplete and people have a limited ability to analyze the available information. However, this does not imply that agents are irrational; they clearly strive to do the right thing. As indicated by many laboratory experiments, the behavior of agents may best be characterized as rule governed (Kahneman et al. 1986).

This also seems to hold in financial markets. Within asset pricing experiments (Smith 1991), for instance, agents adhere to both destabilizing adaptive expectations and stabilizing regressive expectations. Moreover, Taylor and Allen (1992) report that market professionals rely on technical and fundamental analysis to determine their orders. Technical analysis is a prediction method that derives trading signals out of past price movements. As advocated by Murphy (1999), increasing prices indicate buying opportunities while decreasing prices should be interpreted as selling signals. Fundamental analysis, in turn, is concerned with determining the fundamental (intrinsic) value of an asset (Williams 1938). Since fundamentalists believe that the price reverts towards its fundamental value, they are buying (selling) assets that are undervalued (overvalued).

The chartist-fundamentalist approach is based on such observations. For instance, Day and Huang (1990), Brock and Hommes (1998), Lux and Marchesi (2000) and Rosser et al. (2003) show that interactions between speculators who apply technical and fundamental trading rules may create complex (chaotic) price dynamics. Buffeted with dynamic noise, these models have the power to mimic some important stylized facts of financial markets, including bubbles and crashes and excess volatility. Using such a setup, Ehrenstein (2002) and Westerhoff (2003) obtain numerical evidence suggesting that transaction taxes may indeed reduce price fluctuations. However, their high-dimensional nonlinear stochastic models make it quite hard to pin down the causalities acting in the model. Our goal is thus to develop a more simple model of asset price dynamics which enhances our understanding of transaction taxes and allows us to derive some analytical results.

Let us turn to the model. Following Day and Huang (1990), the price of the asset is determined on an order-driven market according to a linear price adjustment function. Such a function describes the relation between the quantity of the asset (e. g. stocks, commodities or foreign currency) bought or sold in a given

time interval and the price change caused by these trades. Hence, the price P of the asset in period $t+1$ is given as

$$(1) \quad P_{t+1} = P_t + a(D_t^C + D_t^F),$$

where a is a positive scale parameter to normalize the order size, and D^C and D^F denote the orders of chartists and fundamentalists, respectively. Accordingly, excess buying drives the price up and excess selling drives it down. Without loss of generality, we set $a=1$.

The asset's fundamental value F is constant. Fundamental analysis suggests buying (selling) when the price is below (above) its fundamental value. The orders of the fundamentalists may thus be formalized as

$$(2) \quad D_t^F = \frac{b}{1+tax}(F - P_t).$$

The first term of the right-hand side of (2) captures the power of fundamental traders, where b denotes a positive reaction coefficient and $tax \geq 0$ is the imposed transaction tax. We assume that the impact of both types of speculators negatively depends on the level of the tax rate (e.g. the agents trade less forcefully). Consequently, the power of fundamentalists decreases with increasing tax rates.

Technical traders submit buying (selling) orders when the price of the asset increases (decreases). Their orders may be expressed as

$$(3) \quad D_t^C = \frac{c}{1+tax}(P_t - P_{t-1}).$$

The amount of the transactions again depends on a reaction coefficient $c > 0$ and on the imposed levy: The larger the tax rate, the weaker the influence of the chartists.

Combining (1)-(3) yields the law of motion of the price

$$(4) \quad P_{t+1} + \left(\frac{b}{1+tax} - \frac{c}{1+tax} - 1 \right) P_t + \frac{c}{1+tax} P_{t-1} = \frac{b}{1+tax} F,$$

which is a second-order linear difference equation.

3. Some analytical results

Now we are ready to present our findings.

Proposition 1: The long-run equilibrium price F , i.e. the fixed point of (4), is independent of transaction taxes.

Inserting $P_{t+1} = P_t = P_{t-1}$ in (4), one obtains $P_t = F$. Therefore, the imposition of a transaction tax does not distort the asset market.

Proposition 2: Transaction taxes always enlarge the area in which the price dynamics of (4) is stable. Put differently, if regulators introduce a transaction tax, some (formerly) unstable price trajectories converge towards F .

Remember that a second-order linear difference equation $S_{t+1} + a_1 S_t + a_2 S_{t-1} = A$ is stable if (i) $1 + a_1 + a_2 > 0$, (ii) $1 - a_1 + a_2 > 0$, and (iii) $1 - a_2 > 0$. Hence, the price evolution is stable if (i) $b > 0$, (ii) $b < 2 + 2tax + 2c$, and (iii) $c < 1 + tax$. Obviously, the larger the tax rate, the weaker the restrictions on the parameters b and c (i. e. they may be larger than without a transaction tax).

As is well known, a second-order linear difference equation generates cycles if $4a_2 > a_1^2$. For our model, this is the case if:

$$c + (1 + tax) - 2\sqrt{c(1 + tax)} < b < c + (1 + tax) + 2\sqrt{c(1 + tax)}.$$

Proposition 3: If the law of motion (4) produces dampened oscillations, the imposition of a transaction tax decreases the amplitude of the cycles.

As demonstrated by Baumol (1961), in the case of dampened fluctuations a decrease of a_2 always yields a lower amplitude. Since $a_2 = c / (1 + tax)$, proposition 3 is obviously true.

4. Numerical examples

Let us illustrate the main consequences of transaction taxes. The top panel of figure 1 presents the evolution of the price after a 1 percent shock (parameter setting: $b=1.05$, $c=0.95$, $F=1$ and $tax=0$). We observe lasting price fluctuations around the fundamental value with decreasing amplitude. Note that there are no fundamental shocks (such as new information). The traders generate their own trading signals: Chartists chase price trends and fundamentalists bet on mean reversion. As a result, volatility is excessive and short-term bubbles occur.

The bottom panel of figure 1 displays a simulation run which is based on the same parameter setting, except that financial market regulators have now imposed a transaction tax of $tax=0.1$. Comparing the two panels reveals that the price dynamics with transaction taxes is less turbulent. The reason is that speculators trade less aggressively if they have to pay transaction taxes. Although a transaction tax does not completely eliminate excessive price fluctuations, it slows down the speculative fever.¹ As demonstrated in the previous section, this holds independently of the parameter setting (which is at least partially surprising since both destabilizing chartists and stabilizing fundamentalists are equally taxed).

5. Conclusions

The fast and hectic trading of speculators may lead to complex endogenous price fluctuations and trigger significant bubbles and crashes. Keynes (1936) therefore proposed the imposition of transaction taxes in financial markets in

¹ Note that within our model the parameter tax is dimensionless, i. e. $tax=0.1$ may stand for a 10 percent or a 0.1 percent transaction tax. Hence, we can only draw qualitative conclusions.

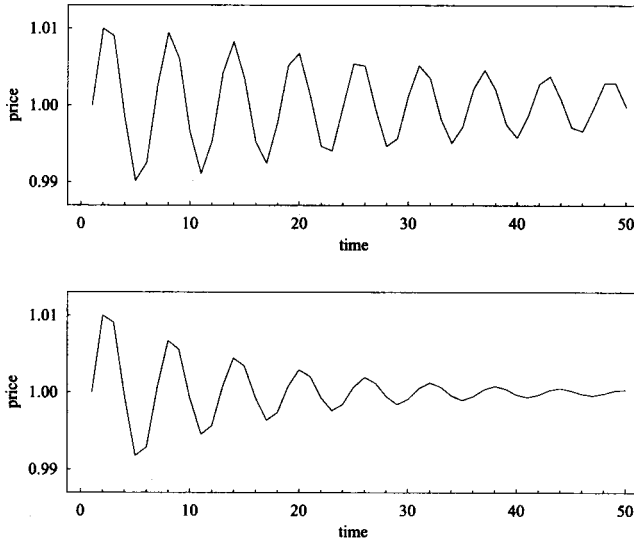


Figure 1: The effectiveness of transaction taxes. The top panel shows the price evolution after a 1 percent shock. Parameter setting: $b=1.05$, $c=0.95$, $F=1$ and $tax=0$. The bottom panel depicts the same, but now $tax=0.1$

order to curb speculation. After Tobin's (1978) suggestion to tax also foreign exchange transactions, this plan gained new momentum. Although this topic is frequently discussed in the popular media, only few theoretical models are concerned with transaction taxes (for a rare macro application see Reitz and Slopek 2003). The goal of this study is to improve our understanding of the workings of transaction taxes. Based on a stylized asset pricing model with boundedly rational technical and fundamental traders we find that the imposition of transaction taxes does not distort long-run equilibrium prices but stabilizes financial markets in the sense that the area in which price fluctuations are stable is enlarged and the amplitude of cycles is reduced. Transaction taxes are obviously ineffective in periods of crisis when prices change violently. But in normal times they reduce the speculative fever and thus may contribute to prevent the next crisis. Given the importance of this policy instrument, more work is, of course, needed. We hope that our note will stimulate research in this area.

Zusammenfassung

Wir entwickeln ein stylisiertes Asset-Pricing-Modell mit interagierenden Chartisten und Fundamentalisten, um die Effizienz von Transaktionssteuern zu untersuchen. Die Lösung des Modells – eine lineare Differenzengleichung zweiter Ordnung – ist in der Lage, zyklische Preisverläufe zu generieren. Wir zeigen

analytisch, dass Transaktionssteuern (1) die langfristigen Gleichgewichtspreise nicht verzerren, (2) den Parameterraum stabiler Preisdynamiken verbreitern, und (3) die Amplitude der Preiszyklen verkleinern. Zusammenfassend scheinen Transaktionssteuern nach Art der Vorschläge von Keynes und Tobin ein effizientes Instrument zu sein, exzessive Preisbewegungen auf spekulativen Märkten zu unterbinden.

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